Diffraction effects on total solar irradiance measurements

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Conclusions

Theoretical diffraction effects are easy to estimate.

Diffraction effects here are small, can be known, and should be taken into account.

We probably know diffraction effects to within a few percent of the effects. Measurement uncertainties limit testing of theory.

Conclusions (this slide)

*Done in collaboration with R.U. Datla and R. Kacker (NIST Statistical Engineering Division)
Diffraction effects on an optical measurement:

Consider paths summed over in Kirchhoff theory:

\[ \Phi = EA \rightarrow \Phi = EAF \]

Diffraction factor

| E = \frac{\Phi}{AF} |

Path length (in the Fresnel, paraxial approximation)

\[ L(\{r_\mu\}) = z_d - z_s + \frac{(x_1 - x_s)^2 + (y_1 - y_s)^2}{2(z_1 - z_s)} + \cdots + \frac{(x_d - x_N)^2 + (y_d - y_N)^2}{2(z_d - z_N)} + \delta L_{foc}(\{r_\mu\}) \]

Iterated Kirchhoff formula gives

\[ u(k, r_s, r_d) = \frac{u_0}{(i\lambda)^N} \int_{A_1 \cdots A_N} d^2r_1 \cdots d^2r_N G(r_s, r_1; k) \cdots G(r_N, r_d; k)e^{ikL(\{r_\mu\})} \]

Free-space, two-point Green’s function

\[ G(r, r'; k) = \frac{\exp(ik |r - r'|)}{|r - r'|} \]
Simplified diffraction calculation
Source-aperture-detector (SAD) Problem

Three main types of SAD geometries:

Overfilled detector:
PMO6, VIRGO, SOVIM, DIARAD, ACRIM, ERBE

Underfilled detector:
SORCE

Undersampled source:
Blackbody calibration
SAD Problem: Diffraction effects on spectral power
Case of a symmetric optical system (Shirley, JOSA A, 2004)

As shown, the detector is underfilled (e.g., SORCE). If the detector perimeter were within the smaller dashed circle, it would be overfilled (e.g., ERBE, ACRIM, PMO6, VIRGO, SOVIM, DIARAD).

From the geometry, introduce parameters,
\[ u = (2\pi R_d^2 / \lambda) (1/d_s + 1/d_d - 1/f), \]
\[ v_s = (2\pi / \lambda) (R_s R_d / d_s), \]
\[ v_d = (2\pi / \lambda) (R_d R_s / d_d), \]
\[ v_0 = \max(v_s, v_d), \]
\[ \sigma = \min(v_s, v_d) / \max(v_s, v_d), \]
\[ w = \min(u/v, \max(u/v)) \]
\[ g = (w + 1/w) / 2 \]

From Wolf, we have the result, for an axial point source,
\[ Y_n(u,v) = \sum_{s=0}^{\infty} (-1)^s (n + 2s)(v/u)^{n+2s} J_{n+2s}(v) \]
\[ Q_{2s}(v) = \sum_{p=0}^{2s} (-1)^p [J_p(v) J_{2s-p}(v) + J_{p+1}(v) J_{2s+1-p}(v)] \]
\[ L_B(v,w) = \sum_{s=0}^{\infty} (-1)^s w^{2s} Q_{2s}(v)/(2s + 1) \]
\[ L_X(v,w) = (4w/v) [Y_1(v/w,v) \cos(gv) + Y_2(v/w,v) \sin(gv)] \]
\[ L(u,v) = w^2 [1 + L_B(v,w)] - L_X(v,w), v < u \]
\[ L(u,v) = 1 - L_B(v,w), v > u \]

We have asymptotic results for \( L_B(v,w) \) and \( L_X(v,w) \) at large \( v \).

We have asymptotic results for \( L_B(v,w) \) and \( L_X(v,w) \) at large \( v \).

Diffraction effects on flux reaching detector:
\[ \Phi_2(\lambda) = C_1 \int_0^1 dx \{(1 + \alpha x)^{-1} [(1 - x^2)(2 + \alpha x)^2 - \sigma^2]\}^{1/2} L(u, v_0(1 + \alpha)) L_2(\lambda) \]
\[ C = 4\pi^3 R_d^4 R_s^2 R_d^2 / [d_s^2 d_d^2 (\lambda v_0)^2] \]

\[ v = v_0(1 + \sigma x) \]

\[ F(\lambda) = \frac{\Phi_2(\lambda)}{\Phi_{\text{ideal}}(\lambda)} \]
SAD Problem: Diffraction effects on total power for a thermal source
Case of a symmetric optical system (Shirley, JOSA A, 2004)

For a thermal source, we have, with \( v = v_0 (1 + \alpha \sigma) \),

\[
\Phi = C \int_0^1 dx (1 + \alpha \sigma)^{-1} \{ (1 - x^2) [(2 + \alpha \sigma)^2 - \sigma^2] \}^{1/2} \int_0^\infty d\lambda \ L(u, \nu) L_2(\lambda)
\]

Introducing

\[
\alpha = v \lambda \quad A = \frac{c_2}{\alpha T} \quad F_X(A, w) = \int_0^\infty \frac{dv v^3}{\exp(Av) - 1} L_X(v, w) \quad F_B(A, w) = \int_0^\infty \frac{dv v^3}{\exp(Av) - 1} L_B(v, w)
\]

we have, for a [small] thermal source,

\[
\int_0^\infty d\lambda \ L(u, \nu) L_2(\lambda) = \frac{\varepsilon c_1}{\pi \alpha^4} \int_0^\infty \frac{dv v^3}{\exp(Av) - 1} \{ w^2 [1 + L_B(v, w)] - L_X(v, w) \}
\]

\[
= \frac{\varepsilon c_1}{\pi \alpha^4} \left[ \frac{6w^2 \zeta(4)}{A^4} + w^2 F_B(A, w) - F_X(A, w) \right].
\]

Asymptotically, we have...

\[
\frac{A^4 F_X(A, w)}{6 \zeta(4)} \approx \left( \frac{16w^6 + 48w^8 + 16w^{10}}{\zeta(4)(1-w^2)} \right) \frac{2 \zeta(3)}{3 \pi \zeta(4)(1-w^2)} A^4 + O\left\{ \exp\left[ - \pi \frac{(w+1/2)}{A} \right] \right\}
\]

\[
\frac{A^4 F_B(A, w)}{6 \zeta(4)} = \left( \frac{1}{24 \pi \zeta(4)(1-w^2)} \right) \left( -1 + 20w^2 + 90w^4 + 20w^6 - w^8 \right) A^4 \log_e(A)
\]

\[
= \frac{(2 + 6 \log_e 2)(-3 + 60w^2 + 270w^4 + 60w^6 - 3w^8) - 768(w^3 + w^5) \log_e[(1+w)/(1-w)] + 9 - 22w^2 - 1354w^4 - 228w^6 + 9w^8}{144 \pi (1-w^2)^5} A^4
\]

\[
= \frac{3 + 8w^2 - 2412w^4 - 51912w^6 - 120750w^8 - 51912w^{10} - 2412w^{12} + 8w^{14} + 3w^{16}}{1536 \pi \zeta(4)(1-w^2)} A^4 \log_e A + O(A^5)
\]

Diffraction effect

\[
F(A, w) - 1 \approx \pm \frac{0.2357 A}{1-w^2}
\]
Results for diffraction effects on TSI measurements:

**NOTE:** The factor \( <F> \) describes the ratio of actual power to ideal power. Therefore, raw measured power is corrected by dividing it by \( <F> \).

\[
E = \frac{\Phi}{A < F >}
\]

For all radiometers, we have:

\[ R_s = 6.75 \times 10^{11} \text{ mm}, d_s = 1.5 \times 10^{14} \text{ mm} \]

Otherwise, diffraction effects are given as follows (assuming \( T=5900 \text{ K} \) for the sun):

<table>
<thead>
<tr>
<th>Radiom.</th>
<th>( R_a ) (mm)</th>
<th>( d_d ) (mm)</th>
<th>( R_d ) (mm)</th>
<th>type</th>
<th>( &lt;F&gt;-1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMO6</td>
<td>4.25</td>
<td>95.4</td>
<td>2.5</td>
<td>a</td>
<td>+0.001280</td>
</tr>
<tr>
<td>VIRGO</td>
<td>4.25</td>
<td>98.5</td>
<td>2.5</td>
<td>a</td>
<td>+0.001323</td>
</tr>
<tr>
<td>SOVIM</td>
<td>4.8</td>
<td>98.5</td>
<td>2.5</td>
<td>a</td>
<td>+0.001025</td>
</tr>
<tr>
<td>DIARAD</td>
<td>6.52</td>
<td>144.</td>
<td>4.0015</td>
<td>a</td>
<td>+0.000833</td>
</tr>
<tr>
<td>ERBE</td>
<td>12.09</td>
<td>100.8</td>
<td>4.039</td>
<td>a</td>
<td>+0.000209</td>
</tr>
<tr>
<td>ACRIM</td>
<td>Baf1</td>
<td>6.6548</td>
<td>150.4696</td>
<td>a</td>
<td>+0.000828</td>
</tr>
<tr>
<td></td>
<td>Baf2</td>
<td>6.3119</td>
<td>76.3524</td>
<td>a</td>
<td>+0.000466</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>+0.001295</td>
</tr>
<tr>
<td>TIM</td>
<td>3.9894</td>
<td>101.6</td>
<td>7.62</td>
<td>b</td>
<td>-0.000418</td>
</tr>
</tbody>
</table>
Estimating systematic uncertainties of diffraction calculations:

**Test case:** Optical set-up of Boivin [Appl. Opt. 17, 3233 (1976); tungsten lamp source, photomultiplier detector, areas defined apertures; Boivin investigated role of non-limiting apertures on throughput (a diffraction effects)].

**Sample set-up:**

![Sample set-up diagram](image)

We use these data to test our diffraction model. In this case, we have

\[
F_{\text{eff}} - 1 = \frac{\int d\lambda \Phi_{0,\lambda}(\lambda) g(\lambda)[F(\lambda) - 1]}{\int d\lambda \Phi_{0,\lambda}(\lambda) g(\lambda)}
\]

where \(g(\lambda)\) accounts for spectral responsivity of detector.
Estimating systematic uncertainties of diffraction calculations:

Comparison of results in test case ($k=1$ meas. unc. shown):
Analysis Method

Assume that relative uncertainty of theoretical diffraction effects is sought. That is, we seek to estimate the typical difference of theoretical diffraction effects from actual diffraction effects. Suppose that we have

\[ \{(F_{m,i}, F_{c,i}), i = 1, N\} \]

Define

\[ Y_{m,i} = \log_e |F_{m,i} - 1|, \quad Y_{c,i} = \log_e |F_{c,i} - 1|, \quad D_i = Y_{m,i} - Y_{c,i} \]

Also have

\[ < D > = N^{-1} \sum_{i=1}^{N} D_i \]

\[ V(D) = (N - 1)^{-1} \sum_{i=1}^{N} (D_i - < D >)^2 \]

\(<D>\) indicates a non-zero average error in \(Y_c\).

\(V(D)\) indicates typical achievable agreement between measurement and theory.
Analysis Method, continued.

Given measurement uncertainties, it is easy to establish the variance,

\[ V(Y_m) = \left\langle u^2(Y_m) \right\rangle \]

One can also calculate the data,

\[ D_i = Y_{m,i} - Y_{c,i} \]

And from this relation, one may deduce

\[ V(D) = V(Y_m) + V(Y_c) \]

THEREFORE, \( u(Y_c) \) can then be deduced from

\[ \left\langle u^2(Y_c) \right\rangle = V(Y_c) = V(D) - V(Y_m) \]
Results of comparison:

In our example, we have

\[ V(D) = 0.00435 = (0.066)^2, \]

\[ <D> = 0.034, \]

indicating no significant non-zero average error in the theory, and

\[ V(Y_m) = 0.00402, \]

Giving

\[ V(Y_c) = 0.000328 = (0.018)^2, \]

or a 1.8 % estimated relative standard uncertainty of \( Y_c \).